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On Multiserver Loss-Delay Queueing System with Priority and No Passing

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ABSTRACT. This investigation deals with a multiserver queueing model in which customers leave the system in the same chronological order in which they arrive. The customers are assumed to arrive in Poisson fashion from two infinite priority and ordinary sources and their service times are identical and exponentially distributed. The priority customers may wait but the ordinary customers are lost if all servers are busy. We have derived analytical expressions for the expected waiting time of customers. Some measures of system performance have been computed and graphs for expected waiting time versus traffic intensity have also been provided.

1. Introduction

Several authors have studied the multiserver queueing systems in which the arriving customers leave the system immediately after getting the required service. In some practical queueing situations, the arriving customers may leave the system in order of their arrival due to some physical restrictions. In this situation no customer can leave system until all customers who have arrived chronologically earlier have also left the system. Examples of such systems include narrow boat lock, a remote border crossing with no parking space ... etc.

A multiserver queueing model under the restriction of departure in chronological order, Poisson input, and exponentially distributed service time was studied by Washburn[1]. Sharma *et al.*[2] extended the same problem for finite waiting space. In many real life queueing problems, the arriving customers may be discouraged by a long queue. Jain *et al.*[3] considered the multiserver queue with no passing which also includes discouragement.

In this paper, we consider a multiserver queueing system in which customers are assumed to arrive from two infinite priority and ordinary sources in Poisson process.

The priority customers are allowed to queue up, while the ordinary customers are lost if all servers are busy. The service times of both type of customers are exponentially distributed. There is a restriction on the departing customers. They can leave the system in the same order in which they arrive. We derive analytical expressions for the expected waiting time.

Our result generalizes those given by Washburn[1]. This includes priority to multiserver loss-delay system with no passing which fulfils the needs of many practical problems.

2. The Model and Analysis

We consider a queueing system in which a group of s servers handle two infinite Poisson streams of priority and ordinary customers with rate λ_1 and λ_2, respectively. The priority customers may wait for service but ordinary customers are lost if all servers are busy. Both types of customers depart from the system in the same chronological order in which they arrive, *i.e.*, nth customer leave the system only when all $(n-1)$ customers arriving earlier have finished their service and have left the system. The service times of both types of customers are exponentially distributed with service rate μ so that the cummulative distribution function (c.d.f.) of service time is of the form

$$F(x) = (1-p) + p\{1 - \exp(-\mu x)\} \quad \text{for } x \geq 0$$
$$0 \leq p \leq 1 \quad , \quad \mu > 0$$

In case of $p = 1$, the c.d.f. of the service times reduces to exponential with service rate μ. Thus the arriving stream is composed of two types of customers. A proportion $(1-p)$ of the customers are ordinary customers and have zero service time and they are lost from the system. The remaining proportion p of customers are priority customers and their service time distribution is exponentially distributed with rate μ.

Let N_t denotes the number of customers in the system at time t and let

$$p_n = \lim_{t \to \infty} \text{Prob.}\ \{N_t = n\}$$

We denote the traffic intensities for priority and ordinary customers by $\rho_1 = \lambda_1 / \mu < s$ and $\rho_2 = \lambda_2 / \mu < s$, respectively.

The steady-state probabilities for the loss delay system with priority is given by (see Saaty 1961[4])

$$p_n = \begin{cases} \dfrac{s-\rho_1}{s-\rho_1+\rho_1 E_s(\rho_1+\rho_2)} \; \dfrac{(\rho_1+\rho_2)^n / n!}{N_s(\rho_1+\rho_2)}, & 0 \leq n \leq s \\[2ex] \dfrac{(s-\rho_1)\, E_s(\rho_1+\rho_2)}{s-\rho_1+\rho_1 E_s(\rho_1+\rho_2)} \cdot \left(\dfrac{\rho_1}{s}\right)^{n-s}, & s \leq n \end{cases} \tag{1}$$

Where, $N_s(\rho) = \sum_{i=0}^{s} (\rho^i / i!)$

$$E_s(\rho) = \frac{\rho^s / s!}{N_s(\rho)}$$

We notice that probability of all s servers busy is given by truncated Poisson distribution $E_s(\rho)$ which is also known as Erlang loss distribution.

Let $E(W_1)$ and $E(W_2)$ be the expected waiting time until departure of customer arriving from priority and ordinary sources, respectively.

We denote $a_n = \int_0^{\infty} \{1 - (1 - \exp(-x)^n\} . dx$

So that $a_n - a_{n-1} = 1/n$ and $a_n = \sum_{j=1}^{n} j - 1$ for $n \geqslant 1$

Following Washburn[1], the expressions for $E(W_1)$ and $E(W_2)$ are given by

$$\mu E(W_1) = a_s + \sum_{n=0}^{s} (a_{n+1} - a_s) . p_n + \sum_{n=s}^{\infty} \frac{n - s + 1}{s} . p_n \quad (2)$$

and , $$\mu E(W_2) = a_{s-1} + \sum_{n=0}^{s-1} (a_n - a_{s-1}) . p_n + \sum_{n=s}^{\infty} \frac{n - s + 1}{s} . p_n \quad (3)$$

On substituting the values of p_n from Equation (1) into Equations (2) and (3), we have

$$\mu E(W_1) = a_s + A_s \sum_{n=0}^{s-1} (a_{n+1} - a_s) \frac{(\rho_1 + \rho_2)^n}{n!} + \left(\frac{B_s}{s}\right)\left(1 - \frac{\rho_1}{s}\right)^{-2} \quad (4)$$

and

$$\mu E(W_2) = a_{s-1} + A_s \sum_{n=0}^{s-1} (a_n - a_s) \frac{(\rho_1 + \rho_2)^n}{n!} + \left(\frac{B_s}{s}\right)\left(1 - \frac{\rho_1}{s!}\right)^{-2} \quad (5)$$

Where

$$A_s = \frac{(s - \rho_1)}{s - \rho_1 + \rho_1 E_s(\rho_1 + \rho_2) . N_2(\rho_1 + \rho_2)}$$

and

$$B_s = \frac{(s - \rho_1) E_s(\rho_1 + \rho_2)}{s - \rho_1 + \rho_1 E_s(\rho_1 + \rho_2)}$$

Let D represents the dimensionless difference between the expected waiting times of two types of customers. Then

$$D = \mu [E(W_1) - E(W_2)]$$

$$= 1/s + A_s \sum_{n=0}^{s-1} \{ (n+1)^{-1} - s^{-1} \} \frac{(\rho_1 + \rho_2)^n}{n!} \tag{6}$$

3. Some Particular Cases

3.1 For s = 1

For single server model, Equations (4) and (6) reduce to

$$\mu E(W_1) = 1 + B_1 (1 - \rho_1)^{-2} \tag{7}$$

where

$$B_1 = \frac{(1 - \rho_1) E_1 (\rho_1 + \rho_2)}{1 - \rho_1 + \rho_1 E_1 (\rho_1 + \rho_2)} \tag{8}$$

and $D = 1$

3.2 For s = 2

In this case Equations (4) and (6) become

$$u E(W_1) = ½ \{ 3 - A_2 + B_2 (1 - \rho_1 / 2)^{-2} \} \tag{9}$$

and

$$D = \frac{1 + A_2}{2} \tag{10}$$

Where

$$A_2 = \frac{2 - \rho_1}{2 - \rho_1 + \rho_1 E_2 (\rho_1 + \rho_2) \,.\, N_2 (\rho_1 + \rho_2)}$$

and

$$B2 = \frac{(2 - \rho_1) E_2 (\rho_1 + \rho_2)}{2 - \rho_1 + \rho_1 E_2 (\rho_: + \rho_2)}$$

3.3 For $\rho_2 = 0$

Its customers arrive from single source, *i.e.*, all customers can wait, we have $\rho_2 = 0$. In this case, Equations (4) and (5) give

$$\mu E(W_1) = \begin{cases} \dfrac{1}{1 - \rho_1} & \text{for } s = 1 \\[2ex] \dfrac{4 + 2\rho_1 - \rho_1^2}{4 - \rho_1^2} & \text{for } s = 2 \end{cases} \tag{11}$$

$$\mu E(W_2) = \begin{cases} 1 & \text{for } s = 1 \\[2ex] \dfrac{2}{2 + \rho_1} & \text{for } s = 2 \end{cases} \tag{12}$$

which also give the solution corresponding to model without loss as discussed by Washburn[1].

4. Numerical Results

In Table 1 and 2, we list for a range of ρ_1, the expected waiting time of priority customers for different values of ρ_2 and $s = 1$ and 2, respectively. Table 3 gives the difference D between the expected waiting time of priority and non-priority customers for $s = 2$.

TABLE 1. $\mu E(W_1)$ for $s = 1$.

ρ_1 \ ρ_2	0	0.3	0.6	0.9
0	1.000	1.231	1.375	1.494
0.1	1.111	1.342	1.486	1.585
0.2	1.250	1.481	1.625	1.724
0.3	1.429	1.659	1.804	1.902
0.4	1.667	1.897	2.041	2.140
0.5	2.000	2.231	2.375	2.474
0.6	2.500	2.731	2.875	2.974
0.7	3.333	3.564	3.708	3.807
0.8	5.000	5.231	5.375	5.474
0.9	10.000	10.231	10.375	10.474

TABLE 2. $\mu E(W_1)$ for $s = 2$.

ρ_1 \ ρ_2	0	0.6	1.2	1.8
0	1.000	1.270	1.452	1.570
0.2	1.101	1.360	1.530	1.640
0.4	1.208	1.458	1.619	1.723
0.6	1.330	1.573	1.725	1.825
0.8	1.476	1.713	1.859	1.954
1.0	1.667	1.900	2.039	2.131
1.2	1.937	2.166	2.302	2.389
1.4	2.373	2.600	2.729	2.814
1.6	3.222	3.444	3.573	3.655
1.8	5.739	6.070	6.082	6.161

5. Discussion

The expected waiting time of priority customers versus traffic intensity ρ_1 when $\rho_2 = 0.3, 0.6, 0.9$ for $s = 1$ and $\rho_2 = 0.6, 1.2, 1.8$ for $s = 2$ are plotted by solid curves in Fig. 1. The broken curves in Fig. 1 show the expected waiting time for system without loss, *i.e.*, for $\rho_2 = 0$. It can be noticed that the expected waiting time increases with traffic intensity for both cases $s = 1$ and 2, and for different values of ρ_2. By increasing servers the expected waiting time decreases which is natural. The expected waiting time changes more rapidly with increase in traffic intensity. For $s = 1, 2$ and different values of ρ_2, the expected waiting time for loss-delay system with priority is

TABLE 3. Difference D, for $s = 2$.

ρ_1 \ ρ_2	0	0.6	1.2	1.8
0	1.000	0.781	0.671	0.613
0.2	0.909	0.732	0.643	0.596
0.4	0.833	0.690	0.619	0.580
0.6	0.769	0.655	0.598	0.567
0.8	0.714	0.624	0.579	0.554
1.0	0.667	0.597	0.562	0.543
1.2	0.625	0.573	0.547	0.533
1.4	0.588	0.552	0.534	0.524
1.6	0.556	0.533	0.521	0.515
1.8	0.526	0.516	0.510	0.507

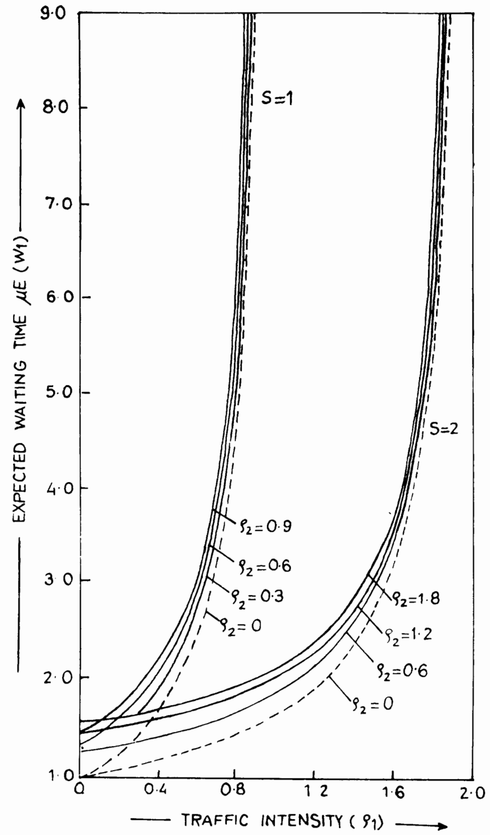

FIG. 1. Expected waiting time $\mu\ E\ (\ W_1\ )$.

greater than the one without loss.

Figure 2 illustrates the difference D versus traffic intensity ρ_1 for $s = 1$ and 2. For system without loss, the difference D is shown by broken curves. It can be seen that as traffic intensity ρ_1 increases, there is a decrease in the value of D for $s = 2$. While for $s = 1$, both systems with and without loss reveal same constant difference $D = 1$.

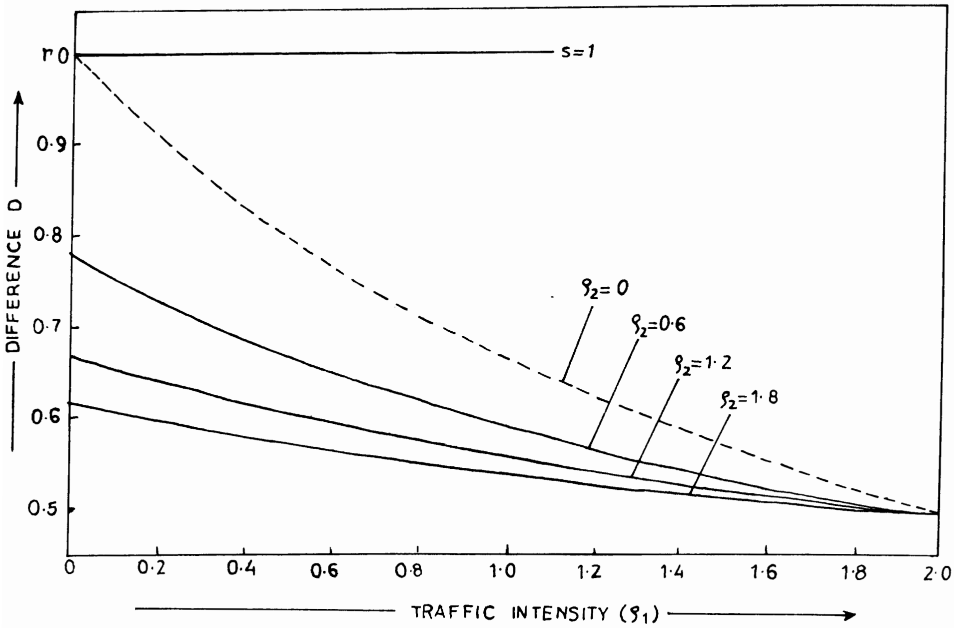

FIG. 2. Difference D.

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حـــول نظـــام طوابـــير متعـــدد الخـــدم ذي خســـارة تأخـــير بأفضليـــة مـــع عـــدم اجتيـــاز

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المستخلص . يعالج هذا البحث نموذج طوابير متعدد الخدم ، فيه يبارح العملاء النظام بنفس الترتيب الزمني الذي وصلوا تبعًا له . ويفترض أن العملاء يصلون بنمط بواسوني من مصدرين لانهائيين ذي أفضلية وعادي ، وأن وقت خدمتهما متماثل وله توزيع أسي . وقد ينتـظر عمـلاء الأفضلية . أمـا العمـلاء العـاديون فإنهم يُفقـدون إذا كان جميع الخدم مشغولون . وقد قمنا باشتقاق صيغ تحليلية لوقت الانتظار المتوقع للعملاء . وقد تم حساب بعض مقـاييس لإنجازية النظام ، وكذلك عمل عرض بياني لوقت الانتظار المتوقع ضد كثافة الحركة المرورية .